

Electron transport in strongly disordered structures.

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Abstract

Using the transfer matrix technique, we investigate the propagation of electron through a two dimensional disordered sample. We find that the spatial distribution of electrons is homogeneous only in the limit of weak disorder (diffusive transport regime). In the limit of very strong disorder, we identify a narrow channel through which the electron propagates from one side of the sample to the opposite side. Even in this limit, we prove the wave character of the electron propagation.

Key words: Localization, wave propagation, conductance, transfer matrix

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1. Introduction

While the propagation of electrons through weakly disordered samples is completely understood [1, 3, 2], the description of electronic transport in the localized regime still opens a new questions. Numerically, it was shown [4, 5] that, contrary to the well-established paradigm, the probability distribution of the logarithm of the conductance is not Gaussian. This was confirmed by recent numerical and analytical analysis [6, 7] and by analytical formulation of the transport in strongly disordered systems [8].

In Ref. [6], the validity of the single parameter scaling was confirmed numerically in the limit of strong disorder. Using the analogy with statistical polymer models, the analytical form for the conductance distribution was derived [7].

Muttalib [8] proposed a generalization of the Dorokhov Mello Pereira Kumar (DMPK) equation [2] to the description of the electron transport in strongly localized systems. Generalized DMPK equation (GDMPK) contains new parameters K_{ab} , which measure the spatial non-homogeneity of electron distribution [9]. Both approximate [9] and numerical [10] solutions of GDMPK equation agree very well with results of numerical simulations [5].

Following the the main idea of GDMPK equation we expect that due to the strong disorder, the spatial distribution

of the electron on the opposite side of the sample is not homogeneous. In this paper, we present the new numerical evidence for this conjecture. With the use of the transfer matrix numerical analysis, we study the spatial distribution of an electron inside the two dimensional disordered sample and show that the electron distribution is homogeneous only in the limit of weak disorder. Stronger disorder causes the formation of continuous cluster of occupied sites inside the sample. This cluster can be interpreted as a trajectory along which electron propagates through the sample. This result agrees with observation of Ref. [7]. We show that the form of this trajectory is very sensitive to the details of random potential. and argue that this sensitivity reflects wave character of the electron propagation [14].

2. Model and method

The two-dimensional Anderson model [11] is defined by the Schrödinger equation

$$E\Psi(\vec{r}) = W\epsilon(\vec{r})\Psi(\vec{r}) + V \sum_{\vec{r}'} \Psi(\vec{r}'). \quad (1)$$

Electron propagates via hopping from the site \vec{r} into the nearest neighbor site \vec{r}' , where $|\vec{r}-\vec{r}'| = a$ and a is the lattice spacing. The size of the system is $L = Na$. The energies $\epsilon(\vec{r})$ are randomly distributed with the Box probability distribution, $P(\epsilon) = 1$ if $|\epsilon| < 1/2$, and $P(\epsilon) = 0$ otherwise. Also, random energies on different sites are statistically independent. The ratio W/V measures the strength of the disorder.

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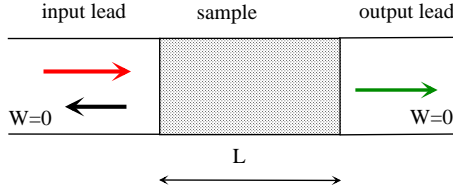


Figure 1: Schematic description of the scattering experiment for the estimation of the transmission. The sample is connected to two semi-infinite leads, represented by tight binding Hamiltonian (1) with zero disorder. Electron is coming from the left. It either propagates through the sample and contributes to the transmission, or is reflected to the left lead.

The disordered sample is connected to two semi infinite, disorder free leads which guide the electron propagation toward and outward the sample (Fig. 1). The incoming electron either propagates through the sample, or is reflected back. The transmission through the sample is determined by the transfer matrix [13]

$$\mathbf{M} = \begin{pmatrix} u & 0 \\ 0 & u^* \end{pmatrix} \begin{pmatrix} \sqrt{1+\lambda} & \sqrt{\lambda} \\ \sqrt{\lambda} & \sqrt{1+\lambda} \end{pmatrix} \begin{pmatrix} v & 0 \\ 0 & v^* \end{pmatrix}, \quad (2)$$

where u, v are $N \times N$ unitary matrices, and λ is a diagonal matrix with positive elements $\lambda_a, a = 1, 2, \dots, N$.

The conductance g is proportional to the transmission T [15],

$$g = \frac{e^2}{h} T, \quad \text{and} \quad T = \sum_a \frac{1}{1 + \lambda_a}. \quad (3)$$

Following GDMPK, we expect that the probability distribution $P(\{\lambda_a\})$ in the insulating regime is influenced by the distribution of an electron on the opposite side of the sample. The last is given by parameters K_{ab} , defined as

$$K_{ab} = \sum_{\alpha} |u_{a\alpha}|^2 |u_{b\alpha}|^2. \quad (4)$$

$K_{ab} = (1 + \delta_{ab})/(N + 1)$ in the diffusive regime [2]. However, if the electron distribution is not homogeneous, then the matrix elements $u_{a\alpha}$ are non-zero only for a small number n of sites ($n \ll N$) and $K_{aa} \sim 1/n \sim 1$.

This conjecture was confirmed by numerical analysis of parameters K_{ab} [9] for the three dimensional Anderson model. Here, we use the transfer matrix technique [16, 17] and the idea of Pichard [13], to visualize the electron distribution inside the disordered sample. For a given sample, we calculate the transmission T , given by Eq. (3). Then, we create an ensemble of N^2 samples, each of them differs from the original one only in the sign of a single random energy $\varepsilon(\vec{r})$, and calculate

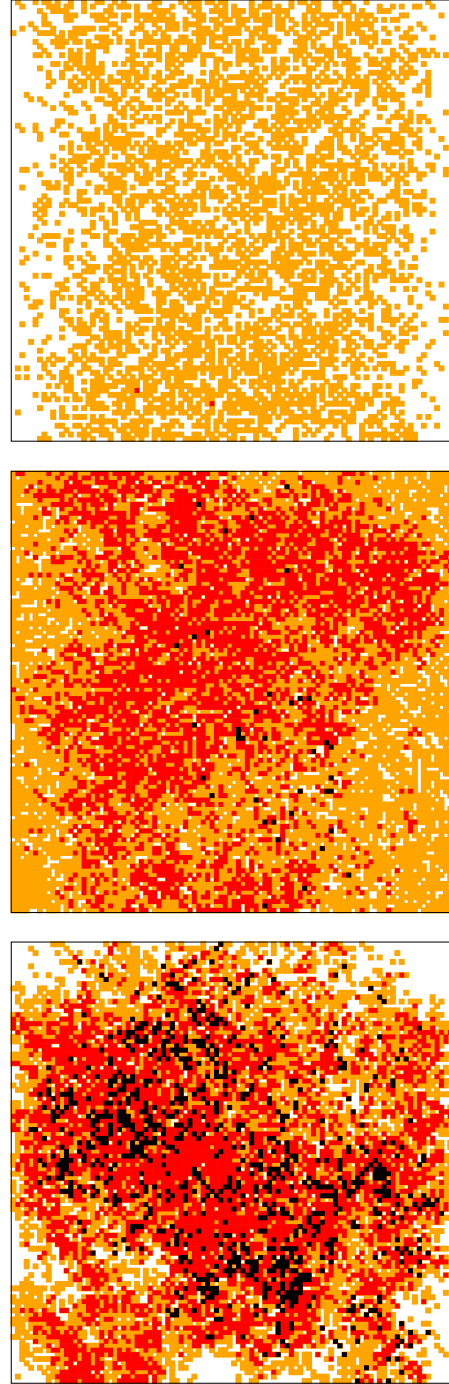


Figure 2: (Color online) Sensitivity of the transmission through the disordered system to the change of the sign of a single random energy ε_0 . Change of the sign of the random energy on orange, red and black sites causes the change of the conductance in more than 1%, 10% and 100%, respectively. The transmission T_0 is 4.998, 0.52 and 0.00084 for the disorder $W/V = 2, 4$ and 6 (from top to bottom). The size of the system is $100a \times 100a$, and the electron propagates from the left side of the sample to the right side.

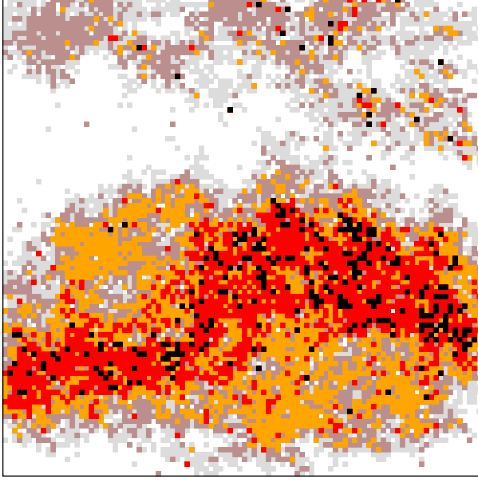


Figure 3: (Color online) The same as in Fig. 2 but with disorder $W/V = 10$. The transmission $T_0 = 9 \times 10^{-15}$. $\langle \ln T \rangle = -33.5$. Shown are also sites where the change of the sign of random energy causes the change of the transmission in 0.01% (gray) and 0.1% (brown).

the transmission $T_{\vec{r}}$ for each sample. The relative difference,

$$\eta(\vec{r}) = \frac{|T_{\vec{r}} - T|}{T} \quad (5)$$

measures the occupancy of the site \vec{r} [13]. Indeed, $\eta(\vec{r})$ is large only if electron resists at the site \vec{r} . If the wave function $|\Psi(\vec{r})|$ at site \vec{r} is small, then the change of the random energy $\varepsilon(\vec{r})$ cannot affect the transmission T so that $\eta(\vec{r})$ is small. The plot of $\eta(\vec{r})$ enables us to identify the highly occupied sites of a given disordered sample.

3. Transmission through disordered sample

3.1. Weak disorder

For weak disorder, $W/V = 2$ (the localization length $\xi \gg L$), the change of only one random energy only negligibly influences the transmission. Top panel of Fig. 2 shows that the transmission T changes only in 1% or even less when the sign of single random energy $\varepsilon(\vec{r})$ changes. Also, it shows that the occupancy of all sample sites is more or less the same. The electron distribution inside the sample is homogeneous, in agreement with the DMPK theory [2], and the random matrix theory of diffusive transport [13]. The lower panels of Fig. 2 demonstrate that the homogeneity of the electron distribution is sensitive to the strength of the disorder.

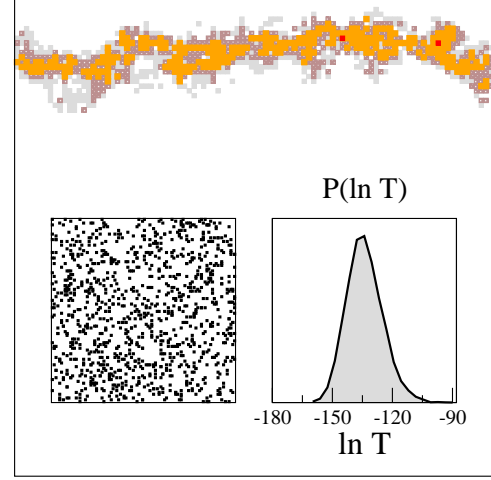


Figure 4: (Color online) The same as in Fig. 2 but for the disorder $W/V = 20$. The transmission of this sample is $\ln T_0 = -96$. Change of the sign of the random energy on gray, brown, orange and red sites causes the change of the logarithm of the transmission in more than 0.01%, 0.1%, 1% and 10% respectively. Left inset shows sites where $|\varepsilon(\vec{r})| < 1$. Right inset shows probability distribution $P(\ln T)$ with the mean value $\langle \ln T \rangle = -133$.

3.2. Strongly localized limit

When the disorder increases, the localization length decreases and becomes smaller than the sample size: $\xi = 5.7a$ ($1.5a$) for disorder $W/V = 10$ ($W/V = 20$, respectively). Although the typical transmission through the strongly disordered sample is small, we can find, thanks to large conductance fluctuations [5], the sample with relatively large transmission.

Figures 3 and 4 show that the spatial electron distribution is not homogeneous inside the strongly disordered systems. Some regions of the sample seem not to be occupied. With increasing disorder, highly occupied sites create a continuous cluster (Fig. 4), which reminds the electron trajectory across the sample [7]. However, even in the case of strong disorder we cannot identify this cluster with the trajectory known from the classical mechanics. Indeed, there are other sites, randomly distributed in other parts of the sample, often located far from the cluster, which influence the transmission as strongly as the sites on the main cluster (Fig. 3). This indicates that the electron propagation is highly sensitive to any change of the realization of the random potential so that the electron wave function is still distributed throughout the entire sample.

The obtained cluster of highly occupied sites cannot be identified with any potential valley or equipotential line in the random potential landscape. To demonstrate

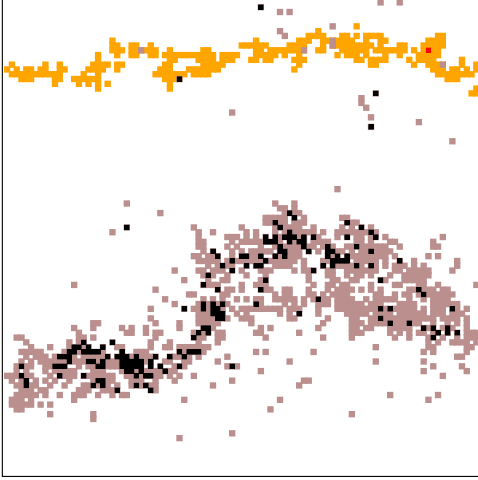


Figure 5: (Color online) Electron “trajectory” through two strongly disordered samples: both samples have the same realization of random energies. They differ only in the amplitude of fluctuations of random potential, defined by Eq. (6). The first sample has $W/V = 10$ and the second has $W/V = 20$. Shown are the lattice sites where the change of the sign of random energy affects the change of the logarithm of the transmission in 1% and 10%. The electron prefers completely different trajectories through these samples. Brown and black sites represent the path of the electron for $W/V = 10$, orange and red sites show the path for $W/V = 20$.

this, we show in inset to Fig. 4 that the spatial profile of the random potential does not indicate any potential valley in the cluster region. Contrary, as expected for the uncorrelated disorder, the spatial distribution of sites with random energy $|\varepsilon| < 1$ is homogeneous. We conclude that the electron trajectory is the effect of quantum interference: the electron comes from the left, inspects the sample, and finds the most convenient spatial channel to propagate.

To support this claim, we consider two disordered samples, which have the same potential profile but differ in amplitude of random fluctuations: $W/V = 10$ for the sample I and $W/V = 20$ for the sample II:

$$\varepsilon(\vec{r})^{II} = 2\varepsilon(\vec{r})^I \quad (6)$$

for all lattice sites \vec{r} . With the use of the above mentioned method, we calculate the highly occupied sites for both samples. There is no reason to expect that the position of these sites changes in the case of classical particle. However, as shown in Fig. 5, the electron chooses completely different trajectories through the two samples. The increase of fluctuations of the random potential causes that electron prefers to transmit through completely different sites than it was in the sample I.

4. Conclusion

We described the propagation of quantum particle through a disordered sample and show how this propagation depends on the strength of the disorder. Our data confirm that the distribution of the electron inside the sample is homogeneous only when the disorder is small. In the limit of strong localization, we find a continuous cluster of preferably visited sites which can be interpreted as a electron trajectory through the sample. This result is consistent with the recent model for the transport through the insulators [7]. We also proved that the obtained trajectory does not contradict the quantum character of electron propagation, and cannot be identify with the trajectory of the classical particle propagating through the sample.

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